

Unification of electromagnetic noise and Luttinger liquid via a quantum dot

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We investigate the effect of dissipation on a small quantum dot (resonant level) tunnel-coupled to a chiral Luttinger liquid (LL) with the LL parameter K . The dissipation stems from the coupling of the dot to an electric environment, being characterized by the resistance R , via Coulomb interactions. We show that this problem can be mapped onto a Caldeira-Leggett model where the (ohmic) bath of harmonic oscillators is governed by the effective dissipation strength $\alpha = (2\tilde{K})^{-1}$ with $\tilde{K}^{-1} = K^{-1} + 2R/R_K$ and $R_K = h/e^2$ the quantum of resistance. Experimental consequences are discussed and the limit $K = 1/2^+$ is thoroughly studied at small R/R_K through the spin-boson-fermion model.

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A quantum dot can be viewed as a simple artificial atom exhibiting charge quantization [1], and its charge can now be measured with a very high accuracy with the aid of an electrometer based on a single-electron transistor [2]. The coupling of the quantum dot to a macroscopic reservoir of electrons inevitably produces quantum charge fluctuations on the dot. This generic phenomenon has been vividly investigated theoretically both in the case of a large metallic box with a very dense spectrum [3] and in the opposite limit of a two-level system [4]. The reservoir of electrons may be a two-dimensional (2D) Fermi-liquid lead [3, 5] or a 1D structure [6, 7] [*e.g.*, a fractional quantum Hall edge state (FQHES) or a quantum wire] where interacting electrons form Tomonaga-Luttinger liquid (LL). Some recent endeavors have been accomplished by Cedraschi *et al.* [4] and by one of us [8] to understand the role of dissipation—coming from the capacitive coupling of the dot to an electric environment with an ohmic resistance—on the charge quantization of a quantum dot, but with the limitation of free electrons in the reservoir lead. In this Letter, we explore dissipation effects on the small quantum dot (resonant level) coupled to a chiral LL (CLL) which has been previously introduced by Furusaki and Matveev [6]. We seek to provide a unified picture of the role of interactions in the one-channel conductor and of the zero-point fluctuations of the electric environment generalizing the case of a single junction [9]. Of interest to us is to understand the nature of the quantum phases emerging through the dissipative mesoscopic structure shown in Fig. 1 and hence to discuss physical implications for the occupation probability on the quantum dot. We highlight that the physics explored here is sufficiently appealing to experimentalists to carry out activities similar to those already existing on superconducting qubits capacitively coupled to lossy transmission lines in GaAs/AlGaAs heterostructures [10].

The quantum dot of interest in Fig. 1 is small enough such that we can only restrict ourselves to the highest-occupied level. This leads to a two-level system [4]. The gate voltage V_g is fixed such that the two states in which the level is occupied ($|1\rangle$) or not ($|0\rangle$) are almost degener-

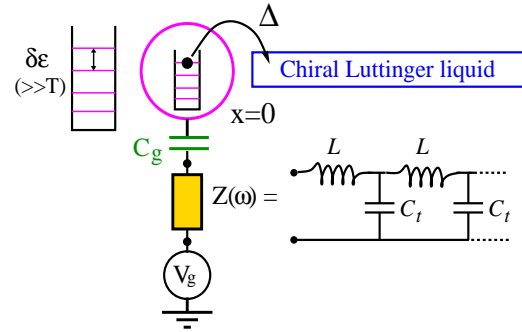


FIG. 1: (Color online) A small dot (with large energy level spacing $\delta\epsilon$) coupled to a CLL (*e.g.*, a quantum wire at the edge $x = 0$). The gate voltage exhibits fluctuations through an impedance $Z(\omega)$ modeled as an LC_t transmission line.

ate. We can thus resort to an orbital spin-1/2 operator, \mathbf{S} , to describe these two states: $S_z = 1/2$ corresponds to the state $|1\rangle$ and $S_z = -1/2$ to $|0\rangle$; S^+ flips the state from $|0\rangle$ to $|1\rangle$, and S^- vice versa. In the presence of electromagnetic noise, the charging Hamiltonian takes the form

$$H_c = \epsilon S_z + (e\hat{Q}_0/C_g)S_z. \quad (1)$$

The detuning ϵ depending on the gate voltage V_g is the energy difference that an electron must overcome if it wants to tunnel between the dot and the lead; here we concentrate on the region close to $\epsilon = 0$ (resonant level). The second term in Eq. (1) arises from the extra capacitive coupling between the dot and the gate voltage fluctuations (the quantum noise) $\delta V_g(t) = \hat{Q}_0/C_g$, with \hat{Q}_0 denoting the charge fluctuation operator on the gate capacitor C_g emerging from the finite impedance $Z(\omega)$ [4, 8]. This term has not been previously considered in Ref. [6]. Akin to Refs. [4] and [11], we find appropriate to model the impedance $Z(\omega)$ in a microscopic fashion through a long dissipative transmission line composed of an infinite collection of LC_t oscillators (Fig. 1); assuming $C_t = C_g$ [12], the latter being governed by the Hamilto-

nian:

$$H_{noise} = \int_0^1 dx \left\{ \frac{\hat{Q}^2(x)}{2C_t} + \frac{\hbar^2}{e^2} \frac{2}{L} \sin^2 \left(\frac{\pi x}{2} \right) \hat{\phi}^2(x) \right\}, \quad (2)$$

where the charge (fluctuation) operator $\hat{Q}(x)$ and the phase operator $\hat{\phi}(x)$ obey the commutation relation $[\hat{\phi}(x), \hat{Q}(y)/e] = i\delta(x-y)$. According to Ref. [11], $\hat{Q}_0 = \sqrt{2} \int_0^1 dx \cos(\pi x/2) \hat{Q}(x)$. At low frequency $\omega \ll \omega_c = 1/(RC_t)$ where the resistance $R = \sqrt{L/C_t}$, the transmission line gives an impedance $Z(\omega) = R/(1+i\omega/\omega_c) \approx R$.

Now, we allow an electron to tunnel between the resonant level and the reservoir lead, *i.e.*, the CLL. Even though the CLL model is the most natural description of the FQHES [13] the charge of the edge excitations depend sensitively on how contacting is done [14]. Therefore, semi-infinite quantum wires — where the wires are coupled to the level only at the edge $x = 0$ [6, 15] as shown in Fig. 1 — represent a more judicious realization of the CLL for our proposal. We only consider the case of spinless electrons which implies that all the electrons have been completely spin-polarized by applying an external magnetic field. The kinetic part of the CLL reads

$$H_{chiral} = \frac{v}{4\pi} \int_{-\infty}^{+\infty} \left(\frac{d\varphi}{dx} \right)^2 dx, \quad (3)$$

where v is the Fermi velocity and the chiral boson field $\varphi(x)$ obey the commutation relations $[\varphi(x), \varphi(y)] = i\pi \text{sgn}(x-y)$. The tunneling processes between the CLL and the level can be described by the Hamiltonian [6]

$$H_{\Delta} = (\Delta/\sqrt{2\pi a}) (e^{i\varphi(0)/\sqrt{K}} S^+ + \text{H.c.}), \quad (4)$$

where a is a short-distance cutoff, Δ the tunneling amplitude, $K < 1$ the LL parameter, and we have exploited the bosonized form $\Psi(0) = (1/\sqrt{2\pi a}) \exp(i\varphi(0)/\sqrt{K})$ of an electron operator $\Psi(0)$ at $x = 0$. In the case of the FQHES, K must be clearly identified as the Landau level filling factor [13]. The total Hamiltonian takes the form $H_{tot} = H_c + H_{noise} + H_{chiral} + H_{\Delta}$. We make the unitary transformation $\mathcal{U}_1 = \exp\{iS_z\hat{\phi}_0\}$, where $\hat{\phi}_0 = \sqrt{2} \int_0^1 dx \cos(\pi x/2) \hat{\phi}(x)$ is the conjugate operator to \hat{Q}_0/e , such that the noise contribution in H_c is completely absorbed in the tunneling part as

$$\bar{H}_{\Delta} = \mathcal{U}_1^\dagger H_{\Delta} \mathcal{U}_1 = (\Delta/\sqrt{2\pi a}) (e^{i\hat{\phi}_0} e^{i\varphi(0)/\sqrt{K}} S^+ + \text{H.c.}). \quad (5)$$

Apparently, the tunneling of an electron between the dot and the CLL must be mediated by excitations in the environmental bosonic modes. It is crucial to bear in mind the large time behavior [16]

$$\mathcal{K}(t) = \langle \hat{\phi}_0(t) \hat{\phi}_0(0) \rangle - \langle \hat{\phi}_0^2 \rangle \simeq -2r \ln(i\omega_c |t|), \quad (6)$$

where $r = R/R_K$ with $R_K = h/e^2 \simeq 25.8k\Omega$ being the quantum of resistance. Let us first establish the renormalization group (RG) equation for the *dimensionless*

tunneling amplitude $\tilde{\Delta} = \Delta/\sqrt{\Lambda}$; $\Lambda = \min(\omega_c, \delta\epsilon)$ is the high-energy cutoff in our model, $\delta\epsilon$ the level spacing on the dot, and we must equate the frequency cutoff of the CLL to $v/a = \Lambda$ (we set $\hbar = k_B = 1$ and v is a dimensionless parameter). Expanding the partition function to second order in Δ and using $\mathcal{K}(t)$ in Eq. (6) give [17]

$$d\tilde{\Delta}/dl = [1 - (2K)^{-1} - r] \tilde{\Delta}, \quad (7)$$

where the RG variable is $l = \ln(\Lambda/T)$ with $T \ll \Lambda$ denoting the temperature. This already allows us to distinguish two different regimes according to the parameter \tilde{K} defined as $1/\tilde{K} = 1/K + 2r$. For $\tilde{K} \ll 1/2$, $\tilde{\Delta}$ is an irrelevant perturbation which means that the physics is dominated by a level being weakly coupled to the CLL, and a perturbation theory in Δ is appropriate. This stands for the “localized” phase where the level is occupied for $\epsilon < 0$ and unoccupied for $\epsilon > 0$, resulting in a jump in the occupation probability $\langle S_z \rangle_\epsilon$ of the level (ϵ is the electron energy relative to the Fermi energy) at $\epsilon = 0$. When $\tilde{K} \gg 1/2$, the level coupling to the CLL is a relevant perturbation that will lift the degeneracy of the ground state at $\epsilon = 0$ and lead to a continuous function of ϵ for $\langle S_z \rangle_\epsilon$ [4, 6]. This is the “delocalized” realm where an electron is resonating back and forth between the CLL and the dot. To get a better description of the strong-coupling fixed point for $\tilde{K} \gg 1/2$ as well as to investigate $\langle S_z \rangle_\epsilon$ on a more quantitative level, we derive an effective Caldeira-Leggett (or ohmic spin-boson) theory [18].

Noting that the level orbital spin only couples to the local CLL mode $\varphi(0)$ and to the “local” noise mode $\hat{\phi}_0$, it is convenient to build the local actions for the modes $\varphi(\tau) = \varphi(x=0, \tau)$ and $\hat{\phi}_0(\tau)$ along the lines of Ref. [15]:

$$S_{chiral}^{loc} = (T/2\pi) \sum_{\omega_n} |\omega_n| \varphi(\omega_n) \varphi(-\omega_n), \quad (8)$$

$$S_{noise}^{loc} = (T/2\pi) \sum_{\omega_n} |\omega_n| (2r)^{-1} \hat{\phi}_0(\omega_n) \hat{\phi}_0(-\omega_n),$$

where ω_n is the bosonic Matsubara frequency. Redefining the fields $\varphi_s = \sqrt{\tilde{K}}(\sqrt{K^{-1}}\varphi + \sqrt{2r}\bar{\phi}_0)$ and $\varphi_a = \sqrt{\tilde{K}}(\sqrt{2r}\varphi - \sqrt{K^{-1}}\bar{\phi}_0)$ where $\bar{\phi}_0 = (1/\sqrt{2r})\hat{\phi}_0$, we find that now the level gets only coupled to the mode φ_s , and therefore it is sufficient to write down the action for φ_s

$$S_{\varphi_s} = S_{\varphi_s}^{loc} + \bar{S}_{\Delta} = \frac{T}{2\pi} \sum_{\omega_n} |\omega_n| \varphi_s(\omega_n) \varphi_s(-\omega_n) - \Delta \sqrt{\Lambda/2\pi v} (e^{i\varphi_s(0)/\sqrt{\tilde{K}}} S^+ + \text{H.c.}). \quad (9)$$

Conceptually, we can visualize S_{φ_s} as the action linked to an Hamiltonian H_{φ_s} modeling a fictive CLL, which is described by an Hamiltonian $H_{chiral}\{\varphi_s\}$ similar to Eq. (3) with a LL parameter \tilde{K} , being coupled to the level. The link with the Caldeira-Leggett model of a two-level system with ohmic dissipation [18] appears when applying the unitary transformation $\mathcal{U}_2 = \exp\{i\varphi_s(0)S_z/\sqrt{\tilde{K}}\}$:

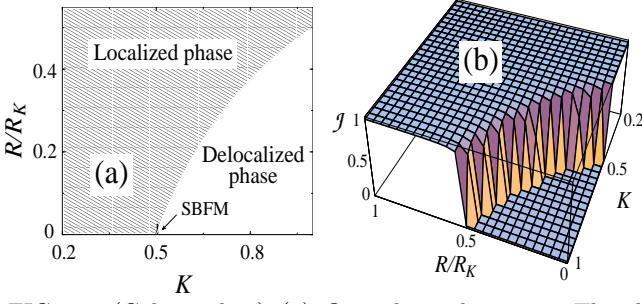


FIG. 2: (Color online) (a) Our phase diagram. The demarcation line is the R_c/R_K versus K curve. This generalizes the (noiseless) situation $R = 0$ of Ref. [6]. (b) $\mathcal{J} = \langle S_z \rangle_{\epsilon=0-} - \langle S_z \rangle_{\epsilon=0+}$ as a function of K and R/R_K . Note the emergence of a conspicuous jump in the entrance of the localized phase. We have set $\frac{\tilde{\Delta}^2}{2\pi v\Lambda} = 0.01 \ll 1$. Below the boundary, $\mathcal{J} = (2\tilde{K})^{1/2} \simeq 1$.

$$\mathcal{U}_2^\dagger H_{\varphi_s} \mathcal{U}_2 = H_{chiral}\{\varphi_s\} + \Delta \sqrt{\frac{\Lambda}{2\pi v}} S_x - \frac{v}{2\sqrt{\tilde{K}}} S_z \frac{d\varphi_s(0)}{dx}. \quad (10)$$

It is well established in this model that a Kosterlitz-Thouless (KT) type quantum phase transition, between a phase where a particle is confined in one of the two levels and another phase where it is delocalized, takes place at the dissipation strength $\alpha = 1$ [18, 19, 20]. Here, $\alpha = 1/(2\tilde{K})$ remarkably consistent with our previous RG result. Note that $\tilde{K} \leq K$. The successful mapping thus immediately allows us to extract a critical value of the resistance in the external circuit R_c which is obviously a function of the strength of the interactions in the CLL:

$$R_c = [1 - (2K)^{-1}] R_K \quad (11)$$

for $K \geq 1/2$, and $R_c = 0$ otherwise. The resulting phase diagram is shown in Fig. 2 (a). The border between the two phases marks the R_c/R_K vs K curve. A significant physical consequence is displayed in Fig. 2 (b) as we discuss below. Fig. 2 is the major result of this Letter.

Now we study the quantity $\mathcal{J} = \langle S_z \rangle_{\epsilon=0-} - \langle S_z \rangle_{\epsilon=0+}$. In the localized phase ($\tilde{K} < 1/2$), we use a second-order perturbation theory in $\tilde{\Delta} \ll 1$ following Ref. [4] to get

$$\mathcal{J} = 1 - \frac{\tilde{\Delta}^2}{2\pi v\Lambda} \frac{\tilde{K}^2}{(1 - \tilde{K})(1 - 2\tilde{K})}. \quad (12)$$

Here, $\tilde{\Delta}$ is a function of T following the RG flow equation (7). Very close to the phase transition region, $\tilde{\Delta} \simeq \Delta$ remains approximately unrenormalized. We plot \mathcal{J} as a function of K and r in Fig. 2 (b) (the upper stair region). Here, the occupation probability $\langle S_z \rangle_\epsilon$ displays a flagrant discontinuity at the Fermi level reflecting the charge quantization on the quantum dot: for $\epsilon < 0$ the level is fully occupied whereas for $\epsilon > 0$ this is empty. From Eq. (9), it becomes transparent that the suppressed noise spectrum in the electric environment at low energy

contributes to reinforcing the vanishing of the tunneling density of states in the CLL at $\epsilon = 0$ [6, 21], $\rho(\epsilon) \propto \epsilon^\mu$ with $\mu = 1/\tilde{K} - 1 = 1/K + 2r - 1 > 1$, which results in a visible localization of particles in the CLL at the edge $x = 0$ (and on the dot) over a large domain of the phase diagram. Just below the boundary, we can generalize the arguments of Ref. [6] to reach a critical value of the jump $\mathcal{J} = (2\tilde{K})^{1/2}$ which is almost one.

In the delocalized phase, the low energy properties of the Caldeira-Leggett model can be identified to those of the anisotropic Kondo model with the Kondo couplings $J_\perp = \Delta\sqrt{\Lambda/2\pi v}$ and $J_z = \frac{v}{2}[1 - 1/(2\tilde{K})^{1/2}]$ [6]. Note that the analogy between the CL problem and the Kondo model was first pointed out in Refs. 19. Not too far from the border (*i.e.*, $\tilde{K} - 1/2 \ll 1$), we are in the weak-coupling Kondo limit where one can easily make use of Bethe-Ansatz results [22]. The emerging Kondo scale Γ corresponds explicitly to the energy scale at which the coupling $\tilde{\Delta}$ gets strongly renormalized in Eq. (7): $\Gamma = \Lambda\tilde{\Delta}^{2\tilde{K}/(2\tilde{K}-1)}$. The Kondo ground state for energies smaller than Γ can be viewed as the complete screening of the orbital spin S_z in the absence of magnetic field ($\epsilon = 0$) implying $\langle S_z \rangle_{\epsilon=0\pm} = 0$; equivalently, the prominent tunneling process smears the quantization of charge on the dot, *i.e.*, makes the occupation probability $\langle S_z \rangle_\epsilon$ continuous around the Fermi level, and thus $\mathcal{J} = 0$ [shown as the lower stair region in Fig. 2 (b)]. This is a clear distinction between the localized and delocalized phases which in principle should be accessible experimentally [2].

The case of $K = 1/2^+$ deserves a special treatment. We still resort to the original Hamiltonian H_{tot} . For $r \ll 1$, the tunneling process in Eq. (4) becomes a marginal operator such that higher order terms in Δ will play a role in the RG flow of Eq. (7). This will slightly modify the value of R_c . We resort to the spin-boson-fermion model [23] to make a zoom into this area denoted by SBFM in Fig. 2 (a). We refermionize H_Δ in Eq. (4):

$$H_\Delta = J_\perp \left(\psi_\downarrow^\dagger(0) \psi_\uparrow(0) S^+ + \text{H.c.} \right), \quad (13)$$

with the dimensionless Kondo coupling $J_\perp = \Delta\sqrt{2\pi v/\Lambda}$ and $\psi_\downarrow^\dagger(x) \psi_\uparrow(x) = [\Lambda/(2\pi v)] \exp(i\sqrt{2}\varphi(x))$. Now, let us rewrite H_c in Eq. (1) exactly like in Ref. [8]:

$$H_c = (\epsilon + \sqrt{r}\Phi) S_z + J_z S_z \left(\psi_\uparrow^\dagger(0) \psi_\uparrow(0) - \psi_\downarrow^\dagger(0) \psi_\downarrow(0) \right), \quad (14)$$

where $\Phi = e\delta V_g/\sqrt{r}$ represents the bosonic variable coupled to the level, and to be fully consistent we have included the Ising part J_z of the Kondo coupling which emerges in the renormalization procedure or which might also be induced by a small Coulomb interaction $u S_z d\varphi(0)/dx$ between electrons in the CLL and the quantum dot. Thus far we have assumed $u = 0$, so at the bare level $J_z = 0$. According to Refs. [8, 11], we can then predict that the delocalized-localized transition will occur at

a *finite* but small R_c ; in the limit of very small J_\perp , we rigorously obtain $R_c = R_K J_\perp$. We finally expect a small jump in the value of R_c at $K = 1/2$ as depicted in Fig. 2 (a). A quantitative discussion for larger values of Δ including a finite Coulomb repulsion u will be addressed elsewhere through a numerical RG approach [24].

The above analysis can also be generalized to the situation where the small dot in Fig. 1 is replaced by a large metallic grain with $\delta\epsilon \rightarrow 0$. We can assume that such a metallic dot is similar to a small quantum wire governed by a Fermi liquid theory, *i.e.*, with a LL parameter $K_g = 1$; charging effects are again taken into account through Eq. (1) but now S_z must be viewed rigorously as a projecting operator acting on the two *charge* states $Q = 0$ and $Q = 1$. This is indeed a valid description when focusing on local physics around $x = 0$ [3, 5]. The tunneling Hamiltonian in Eq. (4) is replaced by $H_{\Delta'} = \Delta' \frac{\Lambda}{2\pi v} (e^{i\varphi(0)/\sqrt{K}} e^{-i\varphi_g(0)} S^+ + h.c.)$, implying that now Δ' is a dimensionless quantity. The operator S^+ flips the charge state $Q = 0$ on the grain to $Q = 1$ [3, 5]. We can introduce the boson fields $\varphi_1 = \sqrt{K_1}(\varphi/\sqrt{K} - \varphi_g)$ and $\varphi_2 = \sqrt{K_1}(\varphi + \varphi_g/\sqrt{K})$ such that the orbital spin is coupled to φ_1 only and the free Hamiltonian for φ_1 takes exactly the same form as in Eq. (3) with the rescaled LL parameter K_1 defined as $1/K_1 = 1 + 1/K$. We can then proceed in a similar manner as before and anticipate a clear shift of the phase boundary in Fig. 2 which is now determined by the equality $2r + 1/K_1 = 2r + 1/K + 1 = 2$. It is crucial to note that for moderate repulsive interactions in the CLL, meaning $K < 1$, the system will be in the localized phase already at $r = 0$, *i.e.*, $R_c = 0$, therefore we infer that the quantum noise has a minor effect on the results. Nevertheless, for a Luttinger exponent $K = 1^-$ which should correctly mimic the situation of a 2D electron reservoir [5], we reproduce the SBFM introduced by one of us in Ref. [8], and especially we recover that $R_c = R_K \Delta'$ for very small tunneling amplitudes Δ' .

To summarize succinctly, we have investigated dissipation effects on a small quantum dot coupled to a CLL which, *e.g.*, can describe a semi-infinite quantum wire as shown in Fig. 1. When the dissipation stems from a lossy transmission line we have been capable of providing a unified description of the role of the interactions in the CLL and of the zero-point fluctuations in the electric environment through the Kondo physics; the derived Caldeira-Leggett theory extending the previous works of Refs. [4, 6, 8] allows us to predict a KT phase transition at $\tilde{K}^{-1} = 2R/R_K + K^{-1} = 2$, separating a localized and a delocalized phase of the dot. Increasing the size of the dot from the nanoscale to the micronscale would strongly reduce the delocalized realm. Recent advanced material technology allows to fabricate clean quantum wires with $0.5 < K < 1$ in GaAs/AlGaAs heterostructures [25]. Other groups have used similar heterostructures to generate a superconducting quantum dot capacitively cou-

pled to a 2D electron gas which serves as the dissipation bath [10]. In particular, the characteristic R can be tuned through changing the 2D electron density. We thus anticipate that the setup shown in Fig. 1 can be built up in such semiconducting heterostructures. The noise induced quantum phase transition we predicted here can thus be tested in experiments, especially by noting that for $K \ll 1$, $R_c \ll R_K = 25.8k\Omega$ which can be easily accessed by experiments.

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- der in $\tilde{\Delta}$ reads $\delta Z \approx -\tilde{\Delta}^2 \Lambda / [(2\pi)^2 a] \int d\tau_1 \int d\tau_2 \mathcal{K}(\tau_1 - \tau_2) (a/(v|\tau_1 - \tau_2|))^{1/K}$ where we must equate $\omega_c = \Lambda = v/a$ and $\tau_i = it_i \gg 1/\Lambda$. δZ must be independent from the energy cutoff Λ , which results in Eq. (7).
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